Microbribery in Group Identification

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Abstract. This paper studies the complexity of two microbribery problems under the model of group identification. In these problems, we are given a subset of distinguished individuals, and the questions are whether these individuals can be made socially qualified or whether they can be made exactly the socially qualified individuals, respectively, by modifying a limited number of entries in the qualifications-profile. For consent rules, the consensus-start-respecting rule, and the liberal-start-respecting rule, we obtain many NP-hardness results and polynomial-time solvability results. We also study the problems in r-profiles where each individual qualifies exactly r individuals.

1 Introduction

In group identification, we are given a profile consisting of a set of individuals each of whom either qualifies (represented by 1) or disqualifies (represented by 0) every individual (including themselves). A subset of individuals, called socially qualified individuals, are identified by a certain social rule based on the valuations of all individuals. It may be relevant to the application where a group of autonomous agents are faced with the problem to select several agents to complete a particular task. Mathematically, group identification can be also regarded as a specific approval-based multiwinner voting model with two specifications: (1) voters and candidates coincide, and (2) there is no restriction on the number of winners.

Since the first work on group identification by Kasher [19], a number of social rules have been proposed, among which, the consent rules, the consensus-start-respecting rule, and the liberal-start-respecting rule have received a considerable amount of attention (see, e.g., [9, 10, 20, 21, 23]). Particularly, these rules have been well characterized by their axiomatic properties which provide significant guidance to evaluate these rules [10, 20, 23]. However, the resistance of these rules to strategic behavior has less been investigated in the literature. So far, only a few papers pertaining to this topic have been published dealing with manipulation, control, and bribery in group identification [13, 14, 30]. In this paper, we continue the line of this research by studying a more fine-grained version of bribery tailored especially for profiles with $0/1$ entries, namely, microbribery in group identification for the aforementioned social rules. Our goal is to provide a more comprehensive guidance on the resistance of these rules to manipulative behaviors. Generally speaking, in microbribery, there is a subset of distinguished candidates and we want to make all of them socially qualified (in a variant, we want these distinguished individuals exactly being the socially qualified individuals) by flipping at most ℓ entries in a given profile. For the aforementioned rules, we establish both NP-hardness results and many polynomial-time solvability results.

1.1 Related Work

First, microbribery was introduced in the context of voting by Faliszewski et al. [15] and has been investigated for several voting rules (for an overview, see the textbooks [6, 22]). In this paper, we are adapting the concept of microbribery to group identification settings. Microbribery has been also considered in the context of lobbying, where a set of voters vote on multiple yes/no issues [5]. The difference in lobbying and group identification is that in lobbying the set of voters is different of the set of issues and a different aggregation procedure is used (mostly majority or some kind of quota rules).

Yang and Dimitrov [30] first studied constructive group control problems from the complexity point of view. [3] After that, Erdélyi, Reger, and Yang [12] extended their work by considering destructive control, and constructive and destructive bribery problems in the model of group identification. In particular, in the bribery problems studied in [12], we are given a subset of individuals and the question is whether we can make these distinguished individuals all socially qualified (constructive) or all nonsocially qualified (destructive) by modifying a limited number of profile rows. In contrast, in microbribery the briber is not allowed to change complete rows with one bribe but is rather limited to changing entries, thus it can only modify at most ℓ entries.

As mentioned above, group identification falls into the category of approval-based multiwinner voting. So, our work is also related to the recent works on the complexity of manipulation, control, and bribery for approval-based multiwinner voting. However, most of these works focus on rules where a fixed number of winners are calculated and the voters and candidates are separated [7, 8, 16, 27, 28]. Complexity of strategic problems for voting rules where the number of winners is not predefined has also been studied recently (see, e.g., [17, 29]), but these rules are different from the ones we shall study in this paper.

In addition, group identification is related to the peer selection problems studied in the literature [1, 3]. The differences are as follows. First, in peer selection, it is often assumed that each individual only evaluates other individuals, and each individual wants to be selected. In addition, similar to standard multiwinner voting, a fixed number of winners are selected. However, in group identification, one is free to disqualify herself/himself and the number of socially qualified individuals it not predefined.

A 3-page extended abstract of this paper appears in the Proceedings of AAMAS 2020 [11]. This version provides the missing proofs in [11].

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[3] A preliminary of their work appeared in the 6th International Workshop on Computational Social Choice (COMSOC 2016).

1.2 Our Contribution

Our main contributions are summarized as follows.

- We first adapt microbribery to the setting of group identification, and study the complexity of microbribery problems under important social rules.
- We introduce and investigate the complexity of the exact variant of microbribery, i.e., after bribery the set of socially qualified individuals has to exactly match the briber's distinguished set of individuals.
- In addition to the general profiles where every individual is allowed to qualify as many individuals as she/he wants, we also study r-profiles where every individual has to qualify exactly r individuals and this restriction should be maintained after bribery.
- For consent rules, the consensus-start-respecting rule, and the liberal-start-respecting rule, we solve the complexity of almost all problems, with only the complexity of microbribery restricted to r-profiles under consent rules remained open for general r (but we have a polynomial-time algorithm for $r = 1$).

Organization. In Section 2, we provide the basic notions to serve our investigations. In Section 3, we formally give the problems which shall be studied in the paper. The following two sections are devoted to the complexity theoretic analysis of the two microbribery problems, respectively. In this two sections, the consent rules, consensus-start-respecting rule, and the liberal-start-respecting rule will be discussed. After this, in Section 6, we study microbribery problems restricted to r-profiles where each individual qualifies exactly r individuals. We conclude our study and point out some interesting topics for future research in Section 7.

2 Preliminaries

In this section we first provide the necessary formal notations followed by an example illustrating the three main social rules.

Let $N := \{a_1, \dots, a_n\}$ denote the set of *individuals* (or *agents*). A *profile* over N is defined as a function $\varphi : N \times N \to \{0, 1\}$, where $\varphi(a_i, a_j) = 1$ means that individual a_i *qualifies* individual a_j, and $\varphi(a_i, a_j) = 0$ means that a_i *disqualifies* a_j. The profile can be represented as a matrix $(\varphi) \in \{0, 1\}^{n\times n}$, where $\varphi_{ij} := \varphi(a_i, a_j)$. In order to aggregate individuals' preferences, we need a *social rule* which is defined as a function f assigning to each pair (φ, N) a subset of individuals $f(\varphi, N) \subseteq N$, referred to as the *socially qualified* individuals with respect to f and φ.

In this paper we are using the following three types of social rules. *Consent rules*, denoted by $f^{(s,t)}$, are specified by the *consent quotas* $s, t \in \mathbb{N}$. For each individual $a_i \in N$,

- if $\varphi(a_i, a_i) = 1$, then $a_i \in f^{(s,t)}(\varphi, N)$ if and only if
$$|\{a_j \in N : \varphi(a_j, a_i) = 1\}| \geq s$$
and
- if $\varphi(a_i, a_i) = 0$, then $a_i \notin f^{(s,t)}(\varphi, N)$ if and only if
$$|\{a_j \in N : \varphi(a_j, a_i) = 0\}| \geq t.$$

The special case where $s = t = 1$ (i.e., an individual is socially qualified if and only if she qualifies herself) is called the *Liberal rule*. Note that the Liberal rule is the only consent rule where each individuals' social qualification only depends on her own assessment and is independent from other agents' opinion.

For the *consensus-start-respecting rule*, denoted by f^{CSR}, we first identify the set of initially qualified individuals $K_0^C(\varphi, N)$ consisting of all the individuals qualified by every individual.

$$K_0^C(\varphi, N) = \{a_i \in N : (\forall a_j \in N)[\varphi(a_j, a_i) = 1]\}.$$

Next, we iteratively expand the set of qualified individuals by adding all the individuals qualified by already socially qualified individuals, until there are no more changes to it. For nonnegative integers k, let $K_k^C(\varphi, N) =$

$$\{a_i \in N : (\exists a_j \in K_{k-1}^C(\varphi, N))[\varphi(a_j, a_i) = 1]\} \cup K_{k-1}^C(\varphi, N).$$

The set of socially qualified candidates is $f^{CSR}(\varphi, N) = K_k^C(\varphi, N)$ for some k such that $K_k^C(\varphi, N) = K_{k+1}^C(\varphi, N)$.

For the *liberal-start-respecting rule*, denoted by f^{LSR}, we again first compute the set of initially qualified individuals by adding all the individuals to $K_0^L(\varphi, N)$ who qualify themselves.

$$K_0^L(\varphi, N) = \{a_i \in N : \varphi(a_i, a_i) = 1\}.$$

Next, we again iteratively expand the set of qualified individuals by adding all the individuals qualified by already socially qualified individuals, until there are no more changes to it. For nonnegative integers k, let $K_k^L(\varphi, N) =$

$$\{a_i \in N : (\exists a_j \in K_{k-1}^L(\varphi, N))[\varphi(a_j, a_i) = 1]\} \cup K_{k-1}^L(\varphi, N).$$

The set of socially qualified candidates is $f^{\text{LSR}}(\varphi, N) = K_k^L(\varphi, N)$ for some k such that $K_k^L(\varphi, N) = K_{k+1}^L(\varphi, N)$.

When φ and N are clear from context, we will simply write K_0^L and K_0^C instead of $K_0^L(\varphi, N)$ and $K_0^C(\varphi, N)$, respectively.

We provide an example to illustrate the workflow of the above social rules.

Example. Let $N = \{a_1, a_2, a_3, a_4\}$ be the set of individuals and let

$$(\varphi) = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

be the profile.

Under the liberal rule (i.e., $s = t = 1$), the set of socially qualified individuals is $f^{(1,1)}(\varphi, N) = \{a_1, a_2\}$, as only these two individuals are qualifying themselves. Under the consent rule $f^{(2,2)}$, the set of socially qualified individuals is $f^{(2,2)}(\varphi, N) = \{a_1, a_3\}$. Note that in contrast to the liberal rule, here individual a_2 is not socially qualified, as there is only one individual qualifying a_2 (namely, a_2 herself/himself), and although a_3 is not qualifying herself, a_3 is socially qualified, as there are in total less than two individuals disqualifying her/him.

Under the consensus-start-respecting rule the initial set of socially qualified individuals is $K_0^C = \{a_1\}$, as this individual is the only one who gets qualified by all individuals. In the following iterations, the set of socially qualified individuals evolves as follows: $K_1^C = \{a_1, a_3\}$, $K_2^C = \{a_1, a_3, a_4\} = K_3^C$, thus the set of socially qualified individuals is $f^{\text{CSR}}(\varphi, N) = K_2^C = \{a_1, a_3, a_4\}$.

Under the liberal-start-respecting rule the initial set of socially qualified individuals is $K_0^L = \{a_1, a_2\}$, as these two individuals are qualifying themselves. In the following iterations, the set of socially qualified individuals changes as follows: $K_1^L = \{a_1, a_2, a_3\}$, $K_2^L = \{a_1, a_2, a_3, a_4\} = K_3^L$, thus the set of socially qualified individuals is $f^{\text{LSR}}(\varphi, N) = K_2^L = \{a_1, a_2, a_3, a_4\}$.

3 Problem Settings

In this section we introduce the formal definitions of two microbribery problems. In the standard bribery setting in group identification introduced by Erdélyi, Reger, and Yang [12], an external agent—the briber—bribes a few individuals and changes their preferences in order to reach its goal of making a preferred set of individuals socially qualified. While in standard bribery once the briber bribes an individual, it can arbitrarily change the individual's preference over all individuals, in microbribery the briber has to pay separately for any flip in the matrix (φ).

For a social rule f, we study the following problems.

f-CONSTRUCTIVE GROUP MICROBRIBERY	
Given:	A 4-tuple (N, φ, S, ℓ) of a set N of n individuals, a profile φ over N, a nonempty subset $S \subseteq N$ with $S \not\subseteq f(\varphi, N)$, and a positive integer ℓ.
Question:	Is there a way to change at most ℓ entries in the matrix φ such that $S \subseteq f(\varphi', N)$ where $\varphi' \in \{0,1\}^{n \times n}$ is the resulting new profile?

We will write f-CGMB, for short. Sometimes a briber's goal is not just getting its preferred individuals socially qualified, but would like to have only exactly those individuals socially qualified.

EXACT-f-CONSTRUCTIVE GROUP MICROBRIBERY	
Given:	A 4-tuple (N, φ, S, ℓ) of a set N of n individuals, a profile φ over N, a nonempty subset $S \subseteq N$ with $S \not\subseteq f(\varphi, N)$, and a positive integer ℓ.
Question:	Is there a way to change at most ℓ entries in the matrix φ such that $S = f(\varphi', N)$ where $\varphi' \in \{0,1\}^{n \times n}$ is the resulting new profile?

We will write EXACT-f-CGMB, for short.

In this paper, we assume the reader is familiar with basic notions in complexity theory and graph theory. We refer to the textbooks [2, 25] for a comprehensive introduction to complexity theory and refer to [4, 26] for graph theory.

It is easy to see that the problems defined above with respect to the aforementioned social rules are clearly in NP. Therefore, for NP-completeness results established in this paper, we only give the proofs for hardness.

4 Microbribery

In this section we investigate the complexity of microbribery in group identification settings under consent, consensus-start-respecting, and liberal-start-respecting rules. Our results are summarized in Table 1.

$f^{(s,t)}$	$O(n^2)$-time solvable (Theorem 1)
f^{LSR}	NP-complete (Theorem 2)
f^{CSR}	NP-complete (Theorem 3)

Table 1. Results for microbribery. Here, n denotes the number of individuals.

Our first result shows that microbribery for consent rules is solvable in polynomial time. In contrast, standard bribery for consent rules is NP-complete for $t \geq 2$ [12]. The difference in the complexity of these problems follows from the fact that for each individual qualification only depends on the corresponding column in the profile φ and while in standard bribery the briber can only manipulate whole rows of the profile φ (thus facing a combinatorial problem that with one bribe the briber can potentially change several individuals' qualification), in microbribery the briber can make changes entry-wise (thus only concentrating on one individual at the time).

Theorem 1 *$f^{(s,t)}$-CGMB can be solved in $O(n^2)$ time for all positive integers s and t, where n denotes the number of individuals.*

PROOF. Given an $f^{(s,t)}$-CGMB instance (N, φ, S, ℓ), we solve it as follows. For each $a \in N$, let $N(a)$ be the set of individuals who qualify a and let $n(a) = |N(a)|$. In addition, let $n = |N|$ be the number of individuals. Let $S' \subseteq S$ denote the set of individuals in S not socially qualified before the microbribery. For each individual $a \in S'$ count

- the minimum number of flips needed to have it socially qualified with $\varphi(a, a) = 1$ defined by $Q_1(a) = s - n(a)$ and
- the minimum number of flips needed to have it socially qualified with $\varphi(a, a) = 0$ defined by $Q_0(a) = (n - t + 1) - n(a)$.

If

$$\sum_{a \in S'} \min(Q_0(a), Q_1(a)) \leq \ell \tag{1}$$

the given instance is a Yes-instance, as the briber can flip enough entries in φ for each $a \in S'$ to make them socially qualify. Otherwise, the given instance is a No-instance.

To see that the algorithm runs in $O(n^2)$ time, observe that both $Q_1(a)$ and $Q_0(a)$ can be calculated in $O(n)$ time for all individuals $a \in S'$. As $|S'| \leq n$, Inequality (1) can be checked in $O(n^2)$ time. $\square$

In contrast to the polynomial-time solvability of the consent rules, we show that microbribery for the two procedural rules are NP-complete. Our results are based on reduction from the following problem.

EXACT COVER BY THREE SETS (X3C)	
Given:	A universe B and a collection $\mathcal{H}$ of subsets of B such that each subset in $\mathcal{H}$ has cardinality 3.
Question:	Does B have an exact set cover, i.e., a subcollection $\mathcal{H}' \subseteq \mathcal{H}$ such that every $b \in B$ occurs in exactly one subset from $\mathcal{H}'$?

It is known that the X3C problem is NP-complete [18].

Theorem 2 *f^{LSR}-CGMB is NP-complete.*

PROOF. Let $(B, \mathcal{H})$ with $B = \{b_1, \ldots, b_{3m}\}$ and $\mathcal{H} = \{H_1, \ldots, H_n\}$ be an instance of X3C. We construct an instance of f^{LSR}-CGMB as follows. Let $N = B \cup \mathcal{H}$. where each H_i, $1 \leq i \leq n$, qualifies exactly the three individuals from B associated to the H_i's in the X3C instance and all the other individuals disqualify every individual. Let the desired set $S = B$ and let $\ell = m$. The instance of f^{LSR}-CGMB is then (N, φ, S, ℓ). Note that in this setting the set of socially qualified individuals is empty since nobody qualified herself.

The construction clearly takes polynomial time. We claim that there is an exact cover $\mathcal{H}'$ of B if and only if individuals in B can be made socially qualified by at most m microbribes.

($\Rightarrow$) Suppose there is an exact cover $\mathcal{H}' \subseteq \mathcal{H}$ of B. Flipping the entries of the corresponding individuals in $\mathcal{H}'$ on evaluating themselves from 0 to 1 makes the individuals in $\mathcal{S}'$ and each individual in B socially qualified.

($\Leftarrow$) Suppose the individuals in B can be qualified by at most m microbribes. With one flip we can qualify no more than three individuals from B (either by qualifying an H_i by herself or via an already qualified individual qualifies such an H_i). Since there are exactly $3m$ individuals in B, each of the m flips had to qualify three different individuals, thus forming an exact cover. □

Based on a similar reduction, we can show the NP-completeness for the consensus-start-respecting rule too.

Theorem 3 f^{CSR}-CGMB *is* NP-*complete.*

PROOF. We again prove NP-hardness via a reduction from the NP-complete problem X3C. Consider a similar construction as in the proof of Theorem 2 with the difference that there is an additional individual a who is initially qualified by everyone (including herself) and who initially qualifies only herself.

We claim that there is an exact cover $\mathcal{H}'$ of B if and only if the individuals in B can be made socially qualified by at most m microbribes.

($\Rightarrow$) Suppose there is an exact cover $\mathcal{H}'$ of B. Flipping the entries of the corresponding individuals in individual a's entries from 0 to 1 makes all the individuals in $\mathcal{H}' \cup B$ socially qualified.

The argumentation for the reverse direction is similar to the one in the proof of Theorem 2. □

5 Exact Microbribery

In this section, we study the exact version of the microbribery problem, where the goal is to make the set S of distinguished individuals exactly the set of socially qualified individuals. Table 2 summarizes our results.

$f^{(s,t)}$	$O(n^2)$-time solvable (Theorem 4)
f^{LSR}	$O(n^3)$-time solvable (Theorem 5)
f^{CSR}	$O(n^4)$-time solvable (Theorem 6)

Table 2. Results for exact microbribery. Here, n denotes the number of individuals.

For consent rules, just as in the case of microbribery, we can provide a polynomial-time algorithm.

Theorem 4 EXACT-$f^{(s,t)}$-CGMB *for all positive integers s and t can be solved in $O(n^2)$ time where n denotes the number of individuals.*

PROOF. A similar algorithm to the one presented in the proof of Theorem 1 works here too with minor difference, that it is not enough to make the individuals in S qualified but the briber also has to make sure no other individual is qualified. This means that the briber also has to consider in its budget the minimum number of flips to disqualify any initially qualified individual $a \in N \setminus S$. The precise algorithm is as follows.

Let (N, φ, S, ℓ) be an instance of EXACT-$f^{(s,t)}$-CGMB. Let $S' \subseteq S$ be the set of individuals in S' who are not socially qualified with respect to N and φ. For each $a \in N$, let $n(a)$ be the number of individuals in N who qualify a with respect to φ. For each individual $a \in S'$, we define $Q_1(a) = s - n(a)$ and let $Q_2(a) = n - t + 1 - n(a)$ as in the proof of Theorem 1. In addition, let $A \subseteq N \setminus S$ be the set of individuals in $N \setminus S$ who are socially qualified with respect to φ. For each $a \in A$, we define

$$\overline{Q}_1(a) = \begin{cases} n(a) - s + 1 & \varphi(a,a) = 1 \\ n(a) - s + 2 & \varphi(a,a) = 0 \end{cases},$$

and

$$\overline{Q}_0(a) = \begin{cases} n(a) - n + t & \varphi(a,a) = 1 \\ n(a) - n + t + 1 & \varphi(a,a) = 0 \end{cases}.$$

Generally speaking, $\overline{Q_1}(a)/\overline{Q_0}(a)$ denotes the minimum number of flips that are needed to make a not socially qualified assuming that a qualifies/disqualifies a in the final profile.

If

$$\sum_{a \in S'} \min\{Q_1(a), Q_0(a)\} + \sum_{a \in A} \min\{\overline{Q_1}(a), \overline{Q_2}(a)\} \leq \ell,$$

we conclude that the given instance is a Yes-instance. Otherwise, we conclude that the instance is a No-instance.

The analysis of the running time of the algorithm is similar to the proof of Theorem 4. □

Next, we study the two procedural rules. Somewhat surprisingly, the exact versions of microbribery for both rules are polynomial-time solvable, standing in contrast to the NP-hardness of their nonexact versions. Roughly speaking, this is because that in the exact versions, we cannot resort to individuals not in S to make individuals in S socially qualified, which restricts the operations we need to consider and significantly shrinks the solution space to explore.

Theorem 5 EXACT-f^{LSR}-CGMB *is solvable in $O(n^3)$ time, where n denotes the number of individuals.*

PROOF. Let (N, φ, S, ℓ) be a given instance of EXACT-f^{LSR}-CGMB. Let $n = |N|$ be the number of individuals. We derive a polynomial-time algorithm to solve the instance as follows.

First, as we want the socially qualified individuals to be exactly those in S, for each individual $a \in N \setminus S$ such that $\varphi(a, a) = 1$, we reset $\varphi(a, a) = 0$ and decrease ℓ by one. Moreover, for all individuals $a \in S$ and all individuals $a' \in N \setminus S$ such that $\varphi(a, a') = 1$, we reset $\varphi(a, a') = 0$ and decrease ℓ accordingly.

Let $S' \subseteq S$ be the set of individuals in S that are not socially qualified so far. If $|S'| \leq \ell$, we can let all individuals in S' qualify themselves so that all of them are socially qualified; we are done. So, in what follows let us assume that $|S'| > \ell$. To proceed, we create an auxiliary directed graph G as follows. We have a vertex for each individual in S'. We have an arc from an individual $a \in S'$ to another individual $a' \in a$ if and only if a qualifies a'. We say a vertex a is reachable from another vertex a' if there is a directed path from a'

to a. In addition, a vertex a is reachable from itself. Moreover, we say a is reachable from a subset of vertices if a is reachable from at least one of the vertex in the subset. We apply the following reduction rule.

Reduction Rule. If there is a strongly connected component C in G such that there does not exist any arc from some individual not in C to someone in C, we do the following:

(1) select an arbitrary individual in C and let the individual qualify herself,
(2) decrease ℓ by one,
(3) remove all individual-vertices from the graph that are reachable from C.

The above reduction is based on the following observation: a strongly connected component C implies that there is a directed Hamiltonian path in the subgraph induced by C. Therefore, if one of the individuals in C becomes socially qualified, all of them become socially qualified, and hence all individuals who are reachable from them become socially qualified.

An exhaustive application of the above rule transforms the graph G into an empty graph. We conclude that the given instance is a Yes-instance if and only if after the above reduction rule is exhaustively used we have $\ell \geq 0$.

It remains to analyze the running time of the algorithm which is dominated by the construction of the auxiliary graph and an exhaustive application of the above reduction rule. The auxiliary graph contains at most n vertices and at most n^2 edges, and can be constructed in $O(n^2)$ time. Each application of the above reduction rule needs to calculate all connected components of the graph which can be done in $O(n + n^2)$ time due to Tarjan's classic algorithm [24]. The three operations in the reduction rule can be finished in $O(n)$ time. So, each application of the above reduction rule takes $O(n + n^2)$ time. As each application of the reduction rule decreases the number individuals by at least one, the reduction rule can be used at most n times. This implies that the running time of the algorithm is bounded by $n \cdot O(n + n^2) = O(n^3)$. □

The algorithm for the consensus-start-respecting rule is analogous.

Theorem 6 EXACT-f^{CSR}-CGMB *is solvable in* $O(n^4)$ *time where* n *denotes the number of individuals.*

PROOF. Let $I = (N, \varphi, S, \ell)$ be a given instance. We derive a polynomial-time algorithm as follows.

First, as we want the socially qualified individuals to be exactly the ones in S, it must be that all individuals in S disqualify all individuals in $N \setminus S$ in the final profile. Therefore, for all $\varphi(a, a') = 1$ where $a \in S$ and $a' \in N \setminus S$, we reset $\varphi(a, a') = 0$, and decrease ℓ accordingly.

Note that in the final profile, at least one of S must be qualified by all individuals in N. We guess such a candidate and check whether a guess leads to a Yes answer. More precisely, we split the given instance into $|S|$ subinstances, each of which takes N, the up-to-date φ, S, ℓ and an individual $a \in S$ as an input, and the question is whether we can flip at most ℓ entries so that S is exactly the set of socially qualified individuals and all individuals qualify a. To solve this subinstance, for all $a' \in N$ such that $\varphi(a', a) = 0$ we reset $\varphi(a', a) = 1$ and decrease ℓ accordingly. Then, let $S' \subseteq S$ be the set of individuals in S that are not socially qualified so far. If $|S'| \leq \ell$, we let a qualify all individuals in S' which makes all individuals in S' socially qualified; we are done. So, let us assume that $|S'| > \ell$. To proceed, we create an auxiliary directed graph G as follows. We have a vertex for each individual in S'. We have an arc from x to y if and only if x qualifies y. We apply the following reduction rule.

Reduction Rule. If there is a strongly connected component C in G such that there does not exist an arc from some individual not in C to someone in C, we do the following:

(1) select an arbitrary individual in C and let a qualify this individual,
(2) decrease ℓ by one,
(3) remove all individuals that are reachable from C.

An exhaustive application of the above rule transforms the graph G into an empty graph. We conclude that the subinstance is a Yes-instance if and only if after the above reduction rule is exhaustively used we have $\ell \geq 0$. Furthermore, the original instance is a Yes-instance if and only if at least one of the subinstances is a Yes-instance.

The analysis of the running time is similar to that in the proof of Theorem 5. However, note that in this case we have at most n subinstances to solve and hence the running time of the algorithm is $n \cdot O(n^3) = O(n^4)$ time. □

6 Microbribery in r-profiles

In this section, we study microbribery problems restricted to r-profiles where every individual has to qualify exactly r individuals. Note that in r-profiles, the bribery limit is always an even number $\ell = 2k$, as the briber has to keep the r-profiles. Our results in this section are summarized in Table 3.

	$r = 1$	$r = 3$	$r \geq 4$
$f^{(s,t)}$	$O(n^2)$ (Thm. 7)		
f^{LSR}	$O(n)$ (Thm. 8)	NP-complete (Thm. 9)	
f^{CSR}	$O(n)$ (Thm. 10)		NP-complete (Thm. 11)

Table 3. Results for microbribery restricted to r-profiles. Here, n denotes the number of individuals.

We start our study with the consent rules.

Theorem 7 *$f^{(s,t)}$-CGMB restricted to r-profiles is polynomial-time solvable for $r = 1$ if at least one of s and t is equal to 1. More precisely, it can be solved in $O(n^2)$ time where n denotes the number of individuals.*

PROOF. Let $(N, \varphi, S \subseteq N, \ell)$ be a given instance of $f^{(s,t)}$-CGMB. Let $n = |N|$ be the number of individuals. Let us consider first the case where $s = 1$. Let $S' \subseteq S$ be the set of individuals in S who are not socially qualified. Clearly, for all individuals $a \in S'$, it holds that $\varphi(a, a) = 0$. We determine that the given instance is a Yes-instance if and only if $|S'| \leq \frac{\ell}{2}$. In fact, to make all individuals in S' socially qualified, the most efficient way is to let everyone in S' qualify herself. This needs to change $2 \cdot |S'|$ entries, two for each $a \in S'$. As S' can be calculated in $O(n^2)$ time, the algorithm runs in $O(n^2)$ time.

Now we consider the case where $t = 1$. We have the following algorithm. Let $S_0 = \{a \in S : \varphi(a, a) = 0\}$ be the set of individuals in S disqualifying themselves which can be calculated in $O(n^2)$

time. For each $a \in S_0$, we let a qualify herself (and disqualify the individual who a qualifies in the original profile) and decrease ℓ by 2. This can be done in $O(n^2)$ time too. Now all individuals in S qualify themselves. Let $S' \subseteq S$ be the set of individuals in S that are socially qualified now. The set S' can be also calculated in $O(n^2)$ time. We maintain a set P of individuals whose presence ensures that all individuals in S' are socially qualified. The set is calculated as follows. Initially let $P = \emptyset$. Then, we consider individuals in S' one by one, and for each $a \in S'$, we add exactly $s-1$ many arbitrarily but fixed individuals in $N \setminus S$ who qualify a into P. Therefore, P' consists of exactly $(s-1) \cdot |S'|$ individuals. This set is again O(n^2)-time computable. Then, we solve the instance as follows. For each individual $a \in S \setminus S'$, let $N(a)$ denote the set of individuals qualifying a now and let $n(a) = |N(a)|$. If $|N \setminus (P \cup S \cup N(a))| \geq s - n(a)$, we select arbitrary but fixed $s - n(a)$ individuals in $N \setminus (P \cup S \cup N(a))$, change them so that each of them only qualifies a, add these individuals together with all individuals in $N(a)$ into P, remove a into S', decrease ℓ by $2(s-n(a))$, and proceed to the next individual in $S \setminus S'$. If, however, $|N \setminus (P \cup S \cup N(a))| < s - n(a)$, we immediately conclude that the given instance is a No-instance. It takes $O(n)$ time to deal with one individual in $S \setminus S'$, and there are at most n individuals to consider. If after doing the above operation we have $S' = S$, we draw the conclusion as follows: if $\ell < 0$, the given instance is a No-instance; otherwise, it is a Yes-instance. With the above hint on the running time of each main step, one can check that the algorithm runs in $O(n^2)$ time. □

Now we study the two procedural rules. We show that the microbribery problem for these two rules are polynomial-time solvable if $r = 1$ but becomes NP-hard when r is little bit bigger. Let us first consider the liberal-start-respecting rule. Observe that when $r = 1$, only individuals qualifying themselves are in the set of qualified individuals. This is to say that for 1-profiles, the liberal-start-respecting rule and the consent rule $f^{(1,1)}$ are identical. Then, from Theorem 7, we know that it can be solved in $O(n^2)$ time. We show that the running time can be improved in this special case.

Theorem 8 f^{LSR}-CGMB *with respect to* 1*-profiles can be solved in linear time with respect to the number of individuals.*

PROOF. Note that for 1-profiles, only individuals qualifying themselves are socially qualified individuals. Therefore, to solve a given f^{LSR}-CGMB instance $(N, \varphi, S, \ell = 2k)$, we check whether the number of initially not socialy socially qualified individuals from S is at most $\ell/2 = k$, i.e.,

$$|\{a \in S : \varphi(a, a) = 0\}| \leq k.$$

If this is the case, the given instance is a Yes-instance; otherwise it is a No-instance. As we need only to check at most $|S|$ entries, the running time of the algorithm is bounded by $O(n)$, where $n = |N|$ is the number of individuals. □

However, for r-profiles where $r \geq 3$, the complexity of the problem changes, as shown in the following theorem.

Theorem 9 f^{LSR}-CGMB *restricted to* r*-profiles is* NP*-complete for any* $r \geq 3$.

PROOF. We only give the proof for 3-profiles. The proof can be extended for any $r \geq 4$ by adding some dummy individuals but keeping the structure of the construction for the case of $r = 3$.

We are giving a reduction from the NP-complete problem X3C. Given the X3C instance $(B, \mathcal{H})$ with $B = \{b_1, \ldots, b_{3m}\}$ and $\mathcal{H} = \{H_1, \ldots, H_n\}$ we construct the following instance $(N, \varphi, S \subseteq N, \ell)$ of f^{LSR}-CGMB. Without loss of generality, let us assume that $m \geq 3$. Let $N = B \cup \mathcal{H} \cup D$ where D is a set of m individuals disjoint from $B \cup \mathcal{H}$. The profile is defined as follows. Each individual in $\mathcal{H}$ qualifies the three individuals from B according to the corresponding three-set. The individuals in B qualify any arbitrary three individuals in D, and every individual in D qualifies herself and any arbitrary two other individuals from D. Furthermore, let $S = B$ and $\ell = 2m$. Note that before microbribery $f(\varphi, N) = D$.

Clearly, the above instance can be created in polynomial time. We claim that there is an exact cover $\mathcal{H}'$ of B if and only if the individuals in B can be made socially qualified by changing at most $2m$ entries such that the resulting new profile is still a 3-profile. Let $(d_1, d_2, \ldots, d_m)$ be an arbitrary but fixed linear order over D.

($\Rightarrow$) Suppose there is an exact cover $\mathcal{H}' \subseteq \mathcal{H}$ of B. Let

$$(H_{\pi(1)}, H_{\pi(2)}, \ldots, H_{\pi(m)})$$

be any arbitrary but fixed linear order over $\mathcal{H}'$. Due to the definition of φ, each individual $d \in D$ qualifies exactly two individuals in $D \setminus \{d\}$. For each $d \in D$, we arbitrarily select one of the two individuals in $D \setminus \{d\}$ qualified by d, and let $d(*)$ be this individual. We do the following changes. For each $d_i \in D$, $1 \leq i \leq m$, we reset $\varphi(d_i, d(*)) = 0$ and reset $\varphi(d_i, H_{\pi(i)}) = 1$. In total, we changed $2m = \ell$ entries. After the changes, every individual of $\mathcal{H}'$ is qualified by one individual in D and hence becomes socially qualified. As $\mathcal{H}'$ is an exact cover, for every $b \in B$, there is an individual $H \in \mathcal{H}'$ such that $\varphi(H, b) = 1$, due to the definition of the profile. Therefore, all individuals in $S = B$ become socially qualified too after the above changes.

($\Leftarrow$) Suppose the individuals in B can be made socially qualified by at most $2m$ microbribes. Any change other than adding an H_i to the set of socially qualified individuals would add at most one individual from B to the set of socially qualified individuals. Since with adding one H_i the briber can at most qualify three individuals from B, and the bribery limit is $2m$ (with only m constructive microbribes), the only way of qualifying all $3m$ individuals from B is if m individuals in D changed an approval from another individual in D to an individual in $\mathcal{H}$, and these m individuals in $\mathcal{H}$ correspond to an exact cover over B. □

Finally, we study the consensus-start-respecting rule. For 1-profiles, we again have a polynomial-time solvability result.

Theorem 10 f^{CSR}-CGMB *restricted to* 1*-profiles can be solved in linear time in the number of individuals.*

PROOF. Let $(N, \varphi, S, \ell = 2k)$ be a given instance. Note that for 1-profiles the set of socially qualified individuals is either empty or has exactly one element. Therefore, if $|S| > 1$, we immediately report that the given instance is No-instance. If $S = \{a\}$, we need only to check whether the number of individuals initially not qualifying a is at most k, i.e.,

$$|\{a' \in N : \varphi(a', a) = 0\}| \leq k.$$

If this is the case, the given instance is a Yes-instance; otherwise, it is a No-instance. As we need only to check at most n entries, the algorithm takes $O(|N|)$ time. □

However, if r increases to 4, the problem becomes intractable.

Theorem 11 *f^{CSR}-CGMB restricted to r-profiles is NP-complete for any $r \geq 4$.*

PROOF. We prove the theorem via a reduction from the X3C problem. Similar to the one in the proof of Theorem 9, we give only the reduction for $r = 4$. The reduction for $r \geq 5$ can be obtained by adding some dummy individuals.

Given an X3C instance $(B, \mathcal{H})$ such that $|B| = 3m$, we construct an instance $(N, \varphi, S \subseteq N, \ell)$ of f^{CSR}-CGMB as follows. Without loss of generality, we assume that $m \geq 4$. First, we define $N = B \cup \mathcal{H} \cup D$ where D is a set of m individuals disjoint from $B \cup \mathcal{H}$. Let $(d_1, d_2, \ldots, d_m)$ be an arbitrary but fixed order over D. The profile φ is defined as follows.

1. We define $\varphi(a, d_1) = 1$ for all individuals $a \in N$ so that d_1 is socially qualified.
2. For each individual $H \in \mathcal{H}$ and each individual $b \in B$, we define $\varphi(H, b) = 1$ if $b \in H$.
3. We let each individual $b \in B$ qualify any three arbitrary individuals in $D \setminus \{d_1\}$.
4. For each $i \in \{1, 2, \ldots, m-3\}$, we define
$$\varphi(d_i, d_{i+1}) = \varphi(d_i, d_{i+2}) = \varphi(d_i, d_{i+3}) = 1.$$
5. We define $\varphi(d_i, d_j) = 1$ for all $i \in \{m-2, m-1, m\}$ and $j \in \{2, m-1, m\}$.
6. For every other $a, a' \in N$ where $\varphi(a, a')$ is not considered above, we define $\varphi(a, a') = 0$.

Note that due to the above definition of φ, every individual in D is qualified by at least two individuals in D. Moreover, all individuals in D are socially qualified. We complete the reduction by setting $S = B$ and $\ell = 2m$.

The above construction clearly can be done in polynomial time. In the following, we show that there is an exact cover $\mathcal{H}'$ of B if and only if the individuals in B can be made socially qualified by at most $2m$ microbribes such that the resulting new profile is still a 4-profile.

($\Rightarrow$) Suppose there is an exact cover $\mathcal{H}'$ of B. Without loss of generality, let $(H_1, H_2, \ldots, H_m)$ be an arbitrary but fixed order of $\mathcal{H}'$. For each d_i, where $i \in \{1, 2, \ldots, m-2\}$, we reset $\varphi(d_i, d_{i+2}) = 0$ and $\varphi(d_i, H_i) = 1$. In addition, we reset $\varphi(d_{m-1}, d_2) = 0$ and $\varphi(d_{m-1}, H_{m-1}) = 1$. Finally, we reset $\varphi(d_m, d_2) = 0$ and $\varphi(d_m, H_m) = 1$. Clearly, we changed exactly $\ell = 2m$ entries. One can check that after doing so, all individuals in D remain socially qualified and, moreover, all individuals in $\mathcal{H}'$ are socially qualified. As $\mathcal{H}'$ is an exact set cover of B, due to the definition of φ (Point 2), every individual $b \in S$ is qualified by exactly one individual $H_i \in \mathcal{H}'$ such that $b \in H_i$. Therefore, all individuals in S are also socially qualified after the changes.

($\Leftarrow$) Suppose the individuals in $S = B$ can be made socially qualified by changing at most $2m$ entries. Any change other than adding an H_i to the set of qualified individuals would add at most one individual from B to the set of qualified individuals. Since with adding one H_i the briber can at most qualify three individuals from B, and the bribery limit is $2m$ (with only m constructive microbribes), the only way of qualifying all $3m$ individuals from B is if $2m$ changes are made in the entries of individuals in D so that the qualified individuals $\mathcal{H}$ by the ones in D cover all individuals in B. □

7 Conclusion

In this paper, we have studied the microbribery and exact microbribery problems in the setting of group identification. For the consent rules, the consensus-start-respecting rule, and the liberal-start-respecting rule, we identified their complexity, offering a guidance of whether these rules are resistant to microbribery behavior. Our results are summarized in Tables 1–3.

For future research, one can study the parameterized complexity of these problems. The problems are clearly fixed-parameter tractable with respect to the number of individuals. An interesting parameter might be the number of distinguished candidates. In addition, it is also important to study approximation algorithms for the optimization versions of the problems studied in the paper. Another interesting operation might be replacing qualified individuals, which means that after bribery, the number of qualified individual by a vote has to remain unchanged. It should be pointed out that microbribery in r-profiles is a special case of this variant, and hence any hardness results established in Section 6 in this paper carry over to this variant of replacing qualified individuals. It should be also mentioned that bribery problems for approval-based multi-winner voting rules with respect to the replacing operation have been studied very recently [16].

Another natural extension of these microbribery models is introducing prices to each flip and the briber would have an overall budget it would be not allowed to exceed. Naturally, as standard microbribery is the unit-price special case of priced-microbribery, the priced-microbribery problems will inherit the hardness results from the standard microbribery cases. Clearly, the polynomial-time algorithms for the consent rules in both microbribery and exact microbribery models can be easily modified in a way that they also work in the priced versions of these models. However, the polynomial-time algorithms for the consensus-start-respecting and the liberal-start-respecting rules in the exact microbribery problems cannot be transformed to the priced versions. We leave these two questions as open problems for future research.

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